REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the revised indexing system printed in Volume 28, Number 128, October 1974, pages 1191–1194.

24 [2.00, 3.10, 3.20, 4.00].—F. B. HILDEBRAND, *Introduction to Numerical Analysis*, 2nd ed., McGraw-Hill Book Co., New York, 1974, xiii + 669 pp., 24 cm. Price $15.50.

The first edition of this well-known introductory text was published in 1956. The present edition preserves not only the basic character of the original work, but also pretty much its content. While many changes have been made, most of them are relatively minor. Among the more substantive additions are new sections on machine errors, recursive computation, Romberg integration, and cubic spline interpolation. Also, the number of problems has increased substantially, from 513 to 670. On the whole, however, the text reflects the state of the art as it existed in the mid-fifties, when the first edition appeared. Sections entitled "Supplementary References", which accompany each chapter, serve to direct the reader to newer developments.

W. G.

25 [2.05.1].—PH. TH. STOL, *Nonlinear Parameter Optimization*, Centre for Agricultural Publishing and Documentation, Wageningen, The Netherlands, 1975, 197 pp., 24 cm. Price 49.40 Dutch guilders.

This interesting book is the author's doctoral thesis, and his abstract which we give below is more accurate than usual.

Nonlinear parameter optimization in least squares was studied from a point of view of differential geometry. Properties of curvilinear coordinates, scale factors and curvature were investigated. Parameters of the condition function were expressed as functions of algorithm parameter to generalize the formulas. The analysis of the convergence process cumulated in the development of procedures that accelerate convergence. Scale factors were used as weights to the differential correction vector to improve the direction of search. A method to correct for curvature, called back projection method, was developed. Use was made of the tangent plane on which the path of search on the fitting surface was projected. Deviations from the original direction were corrected by optimizing the angle of deviation and step factor. The correspondence between rate of convergence and curvature of the path of search was illustrated with an example. A small geodesic curvature at the starting point indicates fast convergence. Curvature properties of the parametric curves appeared to be of more influence than those of the fitting surface. To avoid heavy oscillation of intermediate parameter values a method was developed that required the intermediate points to be the foot of a perpendicular from the terminal point of intermediate observation vectors thus producing paths of controlled approach. Since condition functions may have a complicated structure in that they can be implicit functions, sequential functions or can consist of mathematical models involving alternative functions, it was treated how first derivatives can be calculated and programmed systematically for these functions. Methods introduced were made operational by means of a FORTRAN program. A description of the use of the subprograms and instructions to modify the main program to suit the various algorithms and procedures developed are given in the Appendices.

The strong point of this work is its heavy geometric flavor. Its weakness is in the failure to incorporate good numerical linear algebra into the suggested modifications of the Gauss-Newton algorithm. The author not only forms the normal equations at each step, but he even solves the system by inverting the Gram matrix he should not have formed. The algorithm thus seems inefficient in general and inaccurate for ill-conditioned problems.

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26 [2.05.1].—CHARLES L. LAWSON & RICHARD J. HANSON, *Solving Least Squares Problems*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1974, 340 pp., 24 cm. Price $16.00.

This book is intended both as a text and a reference on solving linear least squares problems. It is written from the numerical analyst's point of view and not only brings together a lot of information previously scattered in research papers, but also contains some original contributions.

The authors evidently have a great deal of hard earned experience from solving least squares problems. The strongest feature of the book is that it covers all aspects of the solution up to a set of field tested portable Fortran programs. For a reader whose immediate concern is with solving problems, it is possible to bypass the first half of the book and pass directly to the last two chapters where the practical aspects are discussed.

The first half of the book develops basic theory and algorithms both for under- and overdetermined systems. Detailed perturbation bounds for the pseudoinverse and the least squares solution are given here. Algorithms based on Householder transformations and the singular value decomposition are then described thoroughly. An algorithm based on sequential Householder reduction for the case when A has a banded structure, is given in a later chapter. Problems when A is more generally sparse are not specially treated.

Two other methods for solving linear least squares problems (normal equations and modified Gram-Schmidt) are briefly described. A more extensive coverage of these and other alternative methods (e.g. the method of Peters and Wilkinson) would have been appropriate and made the book more useful as a textbook. Another topic, which this reviewer thinks should have been included is iterative refinement of a solution.

Linear least squares problems with linear equality or inequality constraints are, however, exhaustively treated. A solution of the problem to minimize $\|Ex - f\|$ subject to $Gx \geqslant h$ is given, which depends on transforming this problem in two steps into a nonnegative least squares problem. This solution gives an elegant modularity in the algorithms for different constrained problems. Unfortunately the transformation described in Chapter 23, Section 5, contains an error, and does not work when the matrix E is rank deficient. Recently in an ICASE report A. K. Cline has shown how to perform a corresponding reduction in the general case.

The last part of the book contains descriptions and ANSI Fortran listings of subroutines for most of the algorithms described earlier in the book. This includes the Householder method, the singular value analysis, the sequential solution of a problem with a banded matrix, the nonnegative least squares solution and the least distance problem. A set of six main programs are also given for validation of these subroutines. The codes can now also be obtained in machine readable form from IMSL.